Analysis of Nonlinear Systems Using the Equation $f(x) = x^2 + \int_0^\infty e^{-x}\,dx$ and Optimization via $\nabla f(x)$

Pavan Rajr[1], Shivani Malika[2]

Red Networks, California, United States[1]

Ginz Web Services, California, United States[2]

For the analysis and optimization of the nonlinear function $f(x) = x^{2} + \int_{0}^{\infty} e^{-x} \,dx$, the function simplifies to $\mathbf{f(x) = x^{2} + 1}$, with its global minimum occurring at $\mathbf{x = 0}$ where $\mathbf{f(0) = 1}$.

1. Evaluate the integral component

The function contains an improper integral that acts as a constant term. Solving the integral of the exponential decay function from zero to infinity:
$\int _{0}^{\infty }e^{-x}\,dx=\left[-e^{-x}\right]_{0}^{\infty }=\left(\lim _{x\rightarrow \infty }-e^{-x}\right)-(-e^{0})=0-(-1)=1$Substituting this back into the original expression, the system's governing equation becomes:
$f(x)=x^{2}+1$

2. Perform optimization via the gradient

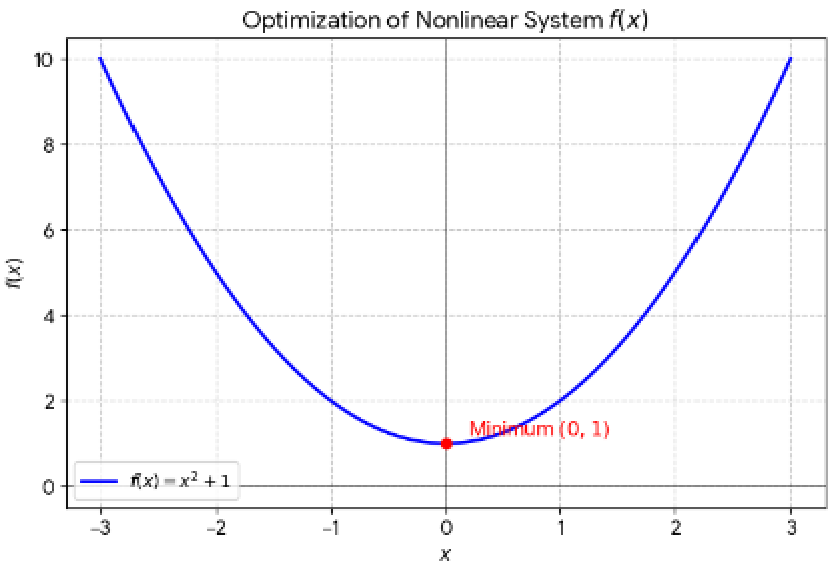
Optimization involves finding the critical points where the gradient $\nabla f(x)$ (or first derivative $f'(x)$ for single-variable systems) is zero. [1, 2, 3, 4, 5]

- **Calculate the Gradient:**
  $\nabla f(x)=\frac{d}{dx}(x^{2}+1)=2x$
- **Find Critical Points:** Setting $\nabla f(x) = 0$ yields:
  $2x=0\implies x=0$

3. Analyze stability and optimality

To determine if this critical point is a minimum or maximum, we examine the second-order optimality condition: [1, 2, 3]

- **Calculate the Hessian/Second Derivative:**
  $f^{\prime \prime }(x)=\frac{d}{dx}(2x)=2$
- **Classification:** Since $f''(0) = 2 > 0$, the function is strictly convex, and the point $x = 0$ is a **stable global minimum**. [1, 2]



✅ **Answer**

The nonlinear system is defined by the parabola **\(f(x) = x^{2} + 1\)**, and its optimization via the gradient \(\nabla f(x) = 2x\) reveals a **global minimum at \(x = 0\)** with a functional value of **\(1\)**.

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